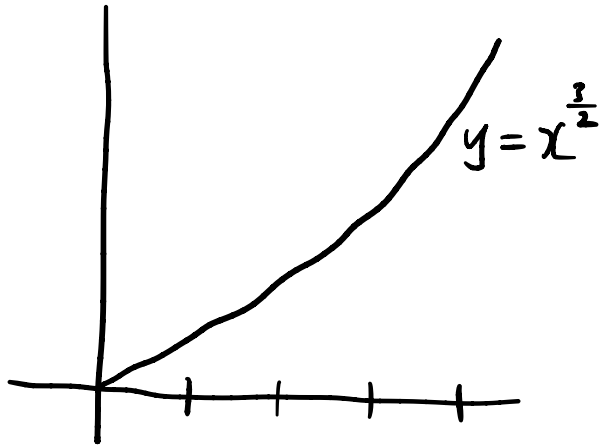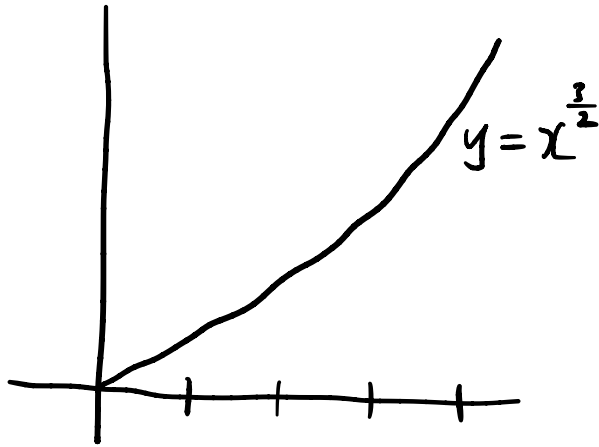Compute the arclength of the curve $y = x^{\frac{3}{2}}$ for $0 \le x \le 4$

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for $0 \leq x \leq 4$

1/9/25

$y = x^{\frac{3}{2}}$

$ds = \sqrt{dx^2 + dy^2}$

$\dfrac{ds}{dx} = \sqrt{1 + \left(\dfrac{dy}{dx}\right)^2}$

$\Rightarrow S = \displaystyle\int \sqrt{1 + f'(x)^2}\, dx$

$y = f(x) = x^{\frac{3}{2}} \Rightarrow f'(x) = \dfrac{3}{2} x^{\frac{1}{2}}$

$S = \displaystyle\int_0^4 \sqrt{1 + \left(\dfrac{3}{2} x^{\frac{1}{2}}\right)^2}\, dx$

$= \displaystyle\int_0^4 \sqrt{1 + \dfrac{9}{4}x}\, dx$

$= \left. \dfrac{\left(1 + \frac{9}{4}x\right)^{\frac{3}{2}}}{\frac{3}{2} \cdot \frac{9}{4}} \right|_0^4$

$S = \left. \dfrac{8}{27}\left(1 + \dfrac{9}{4}x\right)^{\frac{3}{2}} \right|_0^4$

$= \dfrac{8}{27}\left((1+9)^{3/2} - (1)^{3/2}\right)$

$= \dfrac{8}{27}\left(10^{3/2} - 1\right)$